Hydrogen-Accreting Carbon-Oxygen White Dwarfs of Low Mass: Thermal and Chemical Behavior of Burning Shells

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ABSTRACT

Numerical experiments have been performed to investigate the thermal behavior of a cooled down white dwarf of initial mass $M_{\rm WD} = 0.516 M_\odot$ which accretes hydrogen-rich matter with $Z = 0.02$ at the rate $\dot{M} = 10^{-8}\ M_\odot\,{\rm yr}^{-1}$, typical for a recurrent hydrogen shell flash regime. The evolution of the main physical quantities of a model during a pulse cycle is examined in detail. From selected models in the mass range $M_{\rm WD} = 0.52 \div 0.68\ M_\odot$, we derive the borders in the $M_{\rm WD}$ - $\dot{M}$ plane of the steady state accretion regime when hydrogen is burned at a constant rate as rapidly as it is accreted. The physical properties during a hydrogen shell flash in white dwarfs accreting hydrogen-rich matter with metallicities $Z = 0.001$ and $Z = 0.0001$ are also studied. For a fixed accretion rate, a decrease in the metallicity of the accreted matter leads to an increase in the thickness of the hydrogen-rich layer at outburst and a decrease in the hydrogen-burning shell efficiency. In the $M_{\rm WD}$-$\dot{M}$ plane, the borders of the steady state accretion band are critically dependent on the metallicity of the accreted matter: on decreasing the metallicity, the band is shifted to lower accretion rates and its width in $\dot{M}$ is reduced.

Subject headings: stars: novae, cataclysmic variables - stars: accretion - supernovae: general - white dwarfs

1. Introduction

The knowledge of the physical consequences of the accretion of hydrogen-rich matter onto a white dwarf has played an important role in understanding the main properties of several types of eruptive stars such as slow and fast novae and symbiotic stars (e.g., Starrfield 1971, Starrfield, Truran, & Sparks 1978, Sparks, Starrfield, & Truran 1978). In addition, the scenario in which a red giant star transfers mass to its carbon-oxygen (CO) white dwarf companion (Whelan & Iben 1973) is regarded by some as one of the most plausible precursor candidates for Type Ia supernovae (e.g., Hachisu, Kato, & Nomoto 1996). Fairly extensive surveys of the dependence of behavior on white dwarf mass and accretion rate have been conducted (see, e.g., Iben & Tutukov 1996; Cassisi, Iben, & Tornambé 1998 [hereinafter CIT] and references therein). For a low mass CO white dwarf of typical mass in the range $0.5 \div 0.8\ M_{\odot}$, the consequences of the accretion of hydrogen-rich matter of solar metallicity can be summarized as follows:

- for sufficiently large mass-accretion rates (say, $10^{-7}\ M_{\odot}\,\mathrm{yr}^{-1}$ or larger, depending on the white dwarf mass), the accreted layer adopts an expanded configuration similar to that of the envelope of a red giant star;

- for intermediate mass-accretion rates (say, in the range $4 \div 10 \times 10^{-8}\ M_{\odot}\,\mathrm{yr}^{-1}$), the accretor burns hydrogen in a steady state regime at the same rate as it accretes hydrogen;

- for small mass-accretion rates (say, in the range $1 \div 4 \times 10^{-8}\ M_{\odot}\,\mathrm{yr}^{-1}$), the accretor experiences recurrent mild flashes;

- for even smaller mass-accretion rates (say, smaller than $10^{-9}\ M_{\odot}\,\mathrm{yr}^{-1}$), the accretor experiences very strong nova-like hydrogen shell flashes.

Although considerable attention has been paid to models that experience recurrent mild hydrogen-burning flashes (e.g., Paczyński & Żytkow 1978, Iben 1982; José, Hernanz, & Isern 1993, CIT), a deep insight into the evolution of the main physical properties in the accreting models over an outburst-cooling cycle is still missing. Analytical studies of hydrogen-burning shells by Sugimoto & Fujimoto (1978) and by Fujimoto (1982a,b) have established the general properties of the hydrogen-burning shell as a function of the fundamental parameters of the accreting star, but have not provided profiles of structural and chemical variables in the shell itself. Finally, the extant numerical experiments do not explore systematically how the outcome of the accretion process depends on the abundances of heavy elements in the accreted matter.

To investigate in detail the evolution of the main physical characteristics of the hydrogen-burning shell during a flash episode, we have adopted as an initial model a white dwarf of mass 0.516 $M_\odot$ which has accreted matter at the rate $\dot{M} = 10^{-8}\ M_\odot\,\mathrm{yr}^{-1}$ for 7.6×10^5 yr, at which point the interior has been cooled to a temperature of 8.6×10^6 K and the density at the center is 2.56×10^6 g cm^{-3}. For the composition of accreted matter, we have adopted a helium abundance by mass of $Y = 0.28$ and three different metallicities ($Z = 0.02$, 0.001, and 0.0001).

In §2 we discuss the input physics and the assumptions. In §3, the thermal properties of the hydrogen-burning shell are presented and discussed in detail for the Z=0.02 case. In §4, analytical relations in the ($M_{\rm WD}$-$\dot{M}$) plane are obtained for the case Z=0.02 and, in §5, the dependence on metallicity of the evolutionary behavior of the accreting models is analyzed. Conclusions and a brief discussion follow in §6.

2. Input Physics and Assumptions

The initial cold white dwarf model and the accretion experiments have been computed with an updated version of the FRANEC code (Chieffi & Straniero 1989). A detailed discussion of the main differences with respect to the original version of the code is given in CIT. The initial model is the same as that used in CIT and a description of the pre-accretion properties and the first pulse episode can be found in CIT. The main properties of this model are listed in Table 1.

The accretion process is computed on the assumption that the accreted matter and the white dwarf surface have the same specific entropy; that is, it is assumed that all of the energy liberated by matter as it falls onto the surface of the white dwarf is radiated away.

The input physics differs from that used in CIT only in the low temperature opacities: for $Z = 0.02$ and $Z = 0.0001$, we adopt the opacity tables provided by Cox & Stewart (1970a,b) and by Cox & Tabor (1976); for $Z = 0.001$, the opacity values provided by Alexander & Fergusson (1994) have been used. The initial distribution of heavy elements adopted for models of metallicities Z=0.02 and Z=0.0001 is the solar one as given by Ross & Aller (1976); for Z=0.001, the initial distribution is taken from Grevesse (1991).

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3. A Typical Hydrogen-Flash-Driven Pulse Cycle

In this section, we analyze the thermal and nuclear evolution of the hydrogen-burning shell of a model which has experienced enough (about 60) pulses that pulse properties have reached a local asymptotic limit. The model accretes hydrogen-rich matter of solar metallicity (Z=0.02) and its total mass at this point has grown to $M_{\rm WD} \sim 0.5236 M_{\odot}$. We assume that no mass is lost by the accreting star, despite the fact that a relatively nearby companion is required to supply the hydrogen-rich matter to the white dwarf and that, during evolution at high luminosity and low surface temperature, the surface of the star in outburst, in the real analogue, extends in most instances beyond the Roche lobe of the accretor and even beyond the donor star.

The most relevant structural properties of this model are reported in Table 1. The well known evolution in the HR diagram during one complete pulse cycle is reported in Figure 1. Several points of special interest are noted along the track: the positions where flash-driven convection begins (IC) and where it disappears (EC); the positions where maxima and minima of $\Phi_{\rm H} = L_{\rm H}/L_{\rm s}$ and $\Phi_{\rm He} = L_{\rm He}/L_{\rm s}$ occur. Here, $L_{\rm H}$, $L_{\rm He}$, and $L_{\rm s}$ are, respectively, the hydrogen-burning luminosity, the helium-burning luminosity, and the surface luminosity. Several interior characteristics during two passages of the cycle are shown in Figure 2 as a function of the total mass of the accretor.

EDITOR: PLACE FIGURE 1 HERE.

We begin our description at the upper right hand portion of the evolutionary track as the model evolves from red to blue along the high luminosity plateau and $\Phi_{\rm H} \sim 0.994$. In this phase, the release of gravothermal energy $\Phi_{\rm gr} = L_{\rm gr}/L_{\rm s}$ is equal to ~ 0.006, whereas the Helium burning contributes negligibly to the surface luminosity ($\Phi_{\rm He} << 1$). Figure 2a discloses that, along the plateau portion of the evolutionary track, the location in mass of the hydrogen-burning shell (which we define as the point where the maximum rate of energy production via hydrogen-burning is located) moves outwards much more quickly than the total mass grows due to the accretion of fresh matter. In the next section, it will be shown how, during the plateau phase, the relationship between the surface luminosity and the mass of the hydrogen-exhausted core depends on the metallicity of the accreted matter.

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Once the mass of the layer between the hydrogen-burning shell and the surface decreases below a critical value, energy production by hydrogen burning declines rapidly (the reasons for this are discussed by Iben [1982] and the observational consequences are discussed by Iben & Tutukov [1984]). The critical point in the HR diagram is the point of maximum effective temperature labeled BP (for "blue point") in Figure 1.

After reaching the blue point, the hydrogen-burning efficiency plumets and gravothermal energy takes over as the main source of energy (Figure 2d). In the absence of mass accretion, further evolution would be similar to that of a single star after leaving the AGB to become the central star of a planetary nebula; i.e., apart from the onset of crystallization, evolution would not have presented any additional curiosities. However, continuous mass accretion leads to a quite different behavior for the outer layers of the white dwarf relative to that of a non-accreting, cooling white dwarf. In particular, the combined action of the accretion process (growth in mass of the hydrogen-rich envelope) and the contraction of the layers above the hydrogen-burning shell (see Figs. 2e and 2c) relatively quickly brakes the rate of decline of the temperature of the hydrogen-burning shell (Fig. 2b), and leads to a slowly, but continuously, growing hydrogen-burning luminosity (Fig. 2d). In addition, it is worth noticing that the accretion process induces an increase of the evolutionary lifetime in the blue side of the HR loop from the bluest point to the faintest one. In particular along this portion of the cycle the accreting model evolves in a 50% longer time-scale.

The energy delivered by hydrogen burning adds to the rate at which the local temperature in the shell increases until eventually a new hydrogen shell flash occurs. The increase in pressure related to the increase in temperature in the hydrogen-burning shell leads to a rapid expansion of the layers above the shell (see the surface radius increase in Fig. 2f and the decrease in density at the center of the shell in Fig. 2c). In the HR diagram, the model evolves upward as energy diffuses from the burning shell to the surface, and the increase in the surface radius eventually causes the model to evolve to the blue until the plateau portion of the track is reached once again.

Thus, the accretion process modifies the thermal content of the hydrogen-rich layer, producing the physical conditions suitable for a new hydrogen shell flash. An examination of the evolution of the temperature profile in the outer layers of the model and of the profile of the rate of nuclear energy generation in the hydrogen-rich layer shows how this takes place. In Figure 1, circles, triangles, and squares indicate the locations of all models for which profiles are given explicitly in Figures 3 and 4. All models marked by a given symbol in Figure 1 are represented by profiles plotted in a specific panel in Figures 3 and 4. Models marked by solid disks in Figure 1 are represented by the temperature profiles in Figure 3a and the energy-generation profiles in Figure 4a. Models marked by open triangles in Figure 1 are represented by profiles in Figures 3b and 4b. This continues in a counter clockwise fashion with, eventually, the models designated by open squares in Figure 1 being represented by the profiles in Figures 3f and 4f.

EDITOR: PLACE FIGURE 3 HERE.

EDITOR: PLACE FIGURE 4 HERE.

The temperature profile in Figure 3a which has the narrowest “peak” (and acts as a lower envelope of the ensemble of profiles) describes the model designated by the solid disk at smallest surface temperature in Figure 1. Thus, as the model evolves in the HR diagram along the plateau branch from red to blue, thermal energy in the model interior flows inward in the form of a “thermal wave,” heating up hydrogen-free matter. At the same time, thermal energy diffuses outward from the hydrogen-burning shell, which itself propagates outward in mass toward the model surface; the temperature profile between the shell and the surface steepens.

During the next portion of the evolution (the open triangles in Fig. 1 and the profiles in Fig. 3b), thermal energy stored over a fairly large fraction of the outer layers of the model (clearly beyond $M_{\rm r} \sim 0.523\ M_{\odot}$ in Fig. 3b) leaks outward and the drop in temperatures in the hydrogen-burning shell is reflected in a decline in the hydrogen-burning luminosity. Matter interior to $M_{\rm r} \sim 0.522\ M_{\odot}$ is still being heated from above. This behavior continues for the next designated set of models (solid boxes in Fig. 1 and profiles in Fig. 3c), with the "watershed" for thermal energy flow (the place from which energy flows both inward and outward) moving inward. For the following set of models (open circles in Fig. 1 and profiles in Fig. 3d), the energy-flow watershed has moved into interior regions not shown and, until the last few models, when hydrogen burning is beginning to make itself evident again (the bumps near the surface in the temperature profiles in Fig. 3d), cooling prevails over the entire region above $M_{\rm r} \sim 0.521\ M_{\odot}$.

As the new hydrogen-burning shell flash gets underway (the last three open circles and the closed triangles in Fig. 1), a large fraction of the nuclear energy which is released is stored locally, since the time scale on which thermal energy can be transferred is greater than the time scale on which nuclear energy is released. This is true even though matter in the burning region is not degenerate. A convective shell is formed early on in the development of the flash (the point labeled IC in Fig. 1). The mass of the convective shell grows until the shell extends from the base of the burning shell up to photospheric layers. Up to this moment, the evolution of the model has occurred on the nuclear burning time scale. Thereafter, the convective shell (now, really, the convective envelope) cannot accommodate a further increase in its thermal energy content, but must expand to giant dimensions, using up local thermal energy to do the work of expansion against gravity. The readjustment to a new, expanded configuration occurs on the thermal time scale of the envelope. When the energy surplus has been dissipated, (excursion to higher luminosity and lower effective temperature) and the envelope has readjusted to an expanded configuration, the thermal wave begins to propagate inward again (see Fig. 3f). The model is now back to where it began (the profile in Fig 3a with the narrowest peak) .

The profiles of the nuclear energy-generation rate $\epsilon_{\rm n}$ in Figure 4 are also instructive. As it is evident from Figure 4a, along the high luminosity plateau branch, the width in mass of the nuclear energy-generating region increases slightly and the hydrogen-burning luminosity drops as the burning shell works its way toward the surface. During most of the plateau phase, the full CNO cycles are active. By the time the model has attained its maximum effective temperature (Fig. 4b), hydrogen burning rapidly declines in importance as temperatures in the shell (Fig. 3b) decrease. The decline continues along the next portion of the cooling phase (Fig. 4c) as temperatures in the shell continue to decrease (Fig. 3c). An interesting aspect of nuclear burning during the long cooling phase (Figs. 4b, 4c, and 4d) is that it takes place in two distinct regions: a left hand region where the CNO cycles operate and a right hand spike where ^{12}C in freshly accreted fuel is burned into ^{14}N. Along this phase the pp contribution to energy production plays a not negligible role (see the broad secondary peak among the spikes in Fig. 4d).

Ultimately, thanks to the increase in density at the base of the hydrogen-rich layers, heating replaces cooling in the hydrogen-burning layers (Figs. 3d and 4d) and both the CNO cycles and pp-driven $\epsilon_{\rm n}$ profiles begin to increase in height and in breadth. When convection appears, the second $\epsilon_{\rm n}$ peak is "swallowed" by the first (Fig 4d). After the hydrogen-burning luminosity has attained its maximum value and convection begins to recede (Fig. 4e), the mass-width of the region undergoing the strongest hydrogen burning decreases, attaining its minimum width (Fig. 4f) at the same time the model reaches in the

HR diagram the minimum effective temperature.

4. Steady-State Regime for Small White Dwarf Masses

The evolution described in the previous section illustrates the well known result that, for accretion rates smaller than a critical value which depends on white dwarf mass, accreting white dwarfs can be viewed as evolving alternatively in two stable states (a low state and a high state, in the nomenclature of Fujimoto [1982b]), separated by short lived transitional phases. The low state is the cooling phase, when the gravothermal energy source supplies essentially the entire surface luminosity, with hydrogen burning being almost extinguished. The high (or "excited") state is the high luminosity plateau phase during which the hydrogen-burning shell is the main energy source, and the contribution of gravothermal energy is minor.

The hydrogen shell flash acts as the excitation mechanism which induces the transition from the low to the high state. The trigger for the transition is that, when the mass of the accreted layer exceeds a critical value which depends on the accretion rate, the rate of local heating by the hydrogen-burning shell exceeds the rate at which heat can diffuse out, initiating a thermonuclear runaway

The "strength" of a flash (the maximum hydrogen-burning luminosity) and the maximum extension to the red of the evolutionary track during the excited state depend on the accretion rate, in the sense of being greater, the smaller the accretion rate, but the final plateau luminosity during the evolution from red to blue depends only on the mass of the white dwarf. Since the amount of mass accreted between flashes is a quite small fraction of the total mass, the white dwarf mass may be thought of as constant over a large number of cycles; to a very good approximation, the structure of the envelope during the plateau phase does not "remember" the accretion rate preceding the flash which produced it, being sensitive only to the mass of the underlying white dwarf.

The transition from the excited to the low state sets in at the blue point along the evolutionary track in the HR diagram (point BP in Fig. 1). The sudden drop in $\Phi_{\rm H}$ that is shown in Figure 2d is initiated at the blue point. The reason for this second transition can be understood in terms of the dependence on accretion rate of the mass of the hydrogen-rich layer $\Delta M_{\rm H}$ in static models which are forced to burn hydrogen at the same rate as they accrete it (Iben 1982). The blue point in the HR diagram of the locus formed by a sequence of static models of fixed mass but different accretion rate is a bifurcation point (see Fig. 2 in Iben 1982), such that models of successively higher luminosity and lower surface temperature than at the blue point have larger $\Delta M_{\rm H}$, whereas models with successively lower luminosity and surface temperature also have larger $\Delta M_{\rm H}$. If one imagines turning off the accretion rate in any particular model on the upper branch and letting this model evolve statically, $\Delta M_{\rm H}$ in that model would decrease because of nuclear burning and the model would evolve stably to the blue along the sequence, arriving at positions occupied by static steady state models of the same (successively smaller) $\Delta M_{\rm H}$.

If, however, mass accretion were switched off in a model along the lower branch and this model were allowed to evolve statically, the model would evolve upward until it reached the bifurcation point, at which position it would be faced with a quandary. It could not evolve statically in either direction from the bifurcation point.

The implication of these thought experiments is that a real star evolving from red to blue along the plateau branch can do so in a roughly static fashion, with $\Phi_{\rm H} \sim 1$ until, on reaching the bifurcation point, hydrogen burning can no longer control the course of evolution. The static approximation is no longer valid, $\Phi_{\rm H}$ plumets until hydrogen burning is no longer of significance and gravothermal energy has taken over as the prime source of surface luminosity.

These considerations allow us to make use of the properties of accreting models in the quasistatic approximation to estimate, for small white dwarf masses, the lower boundary of the region in the $M_{\rm WD}$-$\dot{M}$ plane where steady state accretion can occur. In Figure 5, the rate $\epsilon_{\rm H}$ of energy generation at the center of the hydrogen-burning shell (panel a) and the rate $\dot{M}_{\rm H}$ at which the hydrogen-burning shell processes matter (panel b) in a model of mass 0.5236 $M_{\odot}$ accreting $Z = 0.02$ matter at the rate 10^{-8} $M_{\odot}\,{\rm yr}^{-1}$ are shown. The position at the maximum surface temperature occurs along the curves in both panels corresponds to the bluest point in the HR diagram of Figure 1, and, there, $\log \dot{M}_{\rm H} = -7.554$

EDITOR: PLACE FIGURE 5 HERE.

In the $M_{\rm WD}$-$\dot{M}_{\rm H-sh}$ plane of Figure 6 are shown curves formed by four additional sets of models for larger white dwarf masses but for the same accretion rate of 10^{-8} $M_{\odot}\,{\rm yr}^{-1}$. The values of $\dot{M}_{\rm H}$ at the blue points for these model tracks and for several others are given in Table 2, where $\Phi_{\rm H}$, $\log T_{\rm e}$ and $\log(L/L_{\odot})$ at the blue points are also given.

EDITOR: PLACE FIGURE 6 HERE.

EDITOR: PLACE TABLE 2 HERE.

A linear fit to the properties of models at the bluest points in the HR diagram gives:

$$\log \dot{M}_{\rm low}(M_{\odot}{\rm yr}^{-1}) = 2.073\frac{M_{\rm WD}}{M_{\odot}} - 8.639, \tag{1}$$

as the lower boundary of the region in which steady state accretion solutions exist over the mass range $0.52 \leq M_{\rm WD}/M_{\odot} \leq 0.68$.

An approximation to the upper boundary of the region where steady state solutions exist can also be derived from the information in Figure 6. The “kink” in each curve near the red end of each curve for the three largest white dwarf masses in Figure 6 actually defines the point beyond which static solutions are of the red giant variety, with the accretion rate being larger than the rate at which nuclear burning consumes fuel (Fujimoto 1982 a,b). In Table 3, the values of $\dot{M}_{\rm H-sh}$ for several models is shown, along with the values of $\Phi_{\rm H}$, $\log T_{\rm e}$ and $\log(L/L_{\odot})$ at the kink. A linear fit between the maximum allowed accretion rate and the WD mass at the kinks provides

$$\log \dot{M}_{\rm high}(M_{\odot}{\rm yr}^{-1}) = 1.512\,\frac{M_{\rm WD}}{M_{\odot}} - 7.800. \tag{2}$$

This line defines the upper boundary of the region in which steady state accretion can occur over the white dwarf mass range $0.52 \leq M_{\rm WD}/M_{\odot} \leq 0.68$.

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The method adopted here to estimate the boundaries of the steady burning zone was introduced by Fujimoto (1982 a,b), who studied the properties of the hydrogen-burning shell in accreting models using an analytical solution for the envelope. He found (see Fig. 4 in Fujimoto 1982b) the border lines to be parallel for white dwarf masses over the range 0.5-1.5 $M_{\odot}$, while our border lines have different slopes. It is probable that our results differ because of the approximations used in the analytical solution. The fact that our lower border in the $M_{\rm WD}$-$\dot{M}_{\rm H}$ plane is steeper than the upper border is consistent with other estimates in the literature (see, e.g. Fig. 7 in Iben & Tutukov 1996 and Fig. 10 in CIT).

5. The Thermal Behavior of the Hydrogen-Burning Shell as a Function of Metallicity

To investigate the dependence on metallicity of the behavior of accreting white dwarfs, we have computed two additional sets of models in which hydrogen-rich matter characterized, respectively, by $Z = 0.001$ and $Z = 0.0001$ is accreted onto the same initial model of mass $0.516 M_\odot$ at the same rate $\dot{M} = 10^{-8}\ M_\odot\,\mathrm{yr}^{-1}$. To make more meaningful the comparison between models accreting mass with different metallicities, we have adopted for all sets of models the same helium abundance: $Y = 0.28$.

In the HR diagram of Figure 7 are shown the paths during one pulse cycle of models of the three different metallicities. In all three cases, the total mass of the model is $M \sim 0.5236 M_\odot$. Several characteristics of the models are given in Table 4. The dependence on the metallicity of various path characteristics can be understood relatively simply. At the very lowest luminosities, all paths converge because the models adopt the essentially metal-independent radius of a cold white dwarf. Because of smaller CNO abundances, the temperatures and densities at the base of the accreted layer (see Table 4) must be larger in models of lower metallicity in order for a CNO cycle thermonuclear runaway to be initiated: in order to achieve larger densities and temperatures, more mass must be accumulated by the lower metallicity models. This is why the time between pulses is larger, the lower the metallicity. During the transition between the low and high states, the radius of the expanding envelope is larger, the larger the mass of the envelope, and this explains why, at any given luminosity, the lower the metallicity, the redder the model. The fact that the reddest point along a path is bluer, the lower the metallicity, can be accounted for as an envelope-opacity effect. Finally, during the plateau phase and during the cooling phase, the fact that, at any luminosity, the model of lower metallicity is redder, is again ascribable to the larger mass of the hydrogen-rich envelope and the consequent larger radius of the envelope.

EDITOR: PLACE TABLE 4 HERE.

EDITOR: PLACE FIGURE 7 HERE.

EDITOR: PLACE FIGURE 8 HERE.

In Figure 8 we have reported for comparison the evolution of the main physical quantities for the hydrogen-burning shell for the cases with $Z = 0.001$ and $Z = 0.0001$. The evolution of $\Phi_{\rm H}$ in panel (d) of this figure demonstrates graphically how the durations of both the high state (plateau phase) and the low state (cooling phase) increase with decreasing metallicity. As we have argued, the plateau phase lasts longer, the lower the metallicity, because the duration of the cooling phase and therefore the mass of hydrogen-rich material accreted between thermonuclear runaways increases with decreasing metallicity. That the amount of mass accreted during the low phase increases with decreasing metallicity is also evident by analyzing panel (a) which show that, the smaller the metallicity, the greater is the amount of fuel burned during the plateau phase. Figure 9 emphasizes this point once again.

EDITOR: PLACE FIGURE 9 HERE.

It is evident from Figure 7, and the discussion in §4, that the band in the $M_{\rm WD} - \dot{M}$ plane where steady state burning solutions exist drops to lower $\dot{M}$ as metallicity decreases. The semianalytical relations obtained in §4 suggest that a $\sim 0.52\ M_\odot$ white dwarf accreting hydrogen-rich matter with $Z = 0.02$ at $8 \times 10^{-8}\ M_\odot\,\mathrm{yr}^{-1}$ settles into a steady state configuration, while, for an accretion rate of $2 \times 10^{-8}\ M_\odot\,\mathrm{yr}^{-1}$, it experiences recurrent mild flashes. To explore quickly the effect of the choice of metallicity on the location of the steady state band, we have calculated models of initial mass $M_{\rm WD} = 0.516 M_\odot$ and accretion rates 2×10^{-8} and $8 \times 10^{-8}\ M_\odot\,\mathrm{yr}^{-1}$, for metallicities of $Z = 0.001$ and $Z = 0.0001$. The model accreting hydrogen at $8 \times 10^{-8}\ M_\odot\,\mathrm{yr}^{-1}$ settles into a red giant configuration after only one pulse, while the model accreting at $\dot{M} = 2 \times 10^{-8}\ M_\odot\,\mathrm{yr}^{-1}$ settles into a steady state accretion configuration after one pulse.

Adopting the method outlined in §4, we have estimated the limits of the steady burning band for the three metallicities when $M_{\rm WD} \simeq 0.5236\ M_\odot$, obtaining the values listed in Table 5.

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We have followed the long term evolution of the low Z models, and from the results (which will be described in detail elsewhere), we have estimated the upper and lower bounds of the steady state band (see Tables 6 and 7, respectively).

EDITOR: PLACE TABLE 6 HERE.

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The upper boundary may be approximated by

$$\log(\dot{M}_{\rm high}(M_\odot \mathrm{yr}^{-1})) = 2.235\ M_{\rm WD}/M_\odot - 8.350 \quad Z = 0.0001 \tag{3}$$

$$\log(\dot{M}_{\rm high}(M_\odot \mathrm{yr}^{-1})) = 1.832\ M_{\rm WD}/M_\odot - 8.043 \quad Z = 0.001 \tag{4}$$

and the lower boundary can be approximated by

$$\log(\dot{M}_{\rm low}(M_{\odot}{\rm yr}^{-1})) = 3.847\ M_{\rm WD}/M_{\odot} - 9.874 \qquad Z = 0.0001 \tag{5}$$

$$\log(\dot{M}_{\rm low}(M_{\odot}{\rm yr}^{-1})) = 2.969\ M_{\rm WD}/M_{\odot} - 9.236 \qquad Z = 0.001 \tag{6}$$

6. Summary and Conclusions

We have investigated and discussed in detail the evolutionary behaviour of a white dwarf accreting hydrogen-rich matter of three different metallicities: $Z = 0.02, 0.001,$ and 0.0001.

An analysis of the evolutionary behavior of several physical characteristics of the models has shown that, for fixed values of M_{WD} and $\dot{M}$, lowering the metallicity causes the recurrence period to become longer because, in order to achieve the larger temperatures and densities necessary to offset the reduction of CNO catalysts in the accreted matter, the thickness of the hydrogen-rich accreted layer must increase.

For the steady-state burning regime, we have been able to derive borders in the M_{WD}-$\dot{M}$ plane as they depend on the metal content of the accreted matter. In agreement with earlier estimates, we find that the area of the region in this plane in which steady-state burning takes place becomes narrower as the white dwarf mass is increased. In addition, the location and the extension of the steady-state burning regime have been found to depend critically on the metallicity of the accreted matter, as shown clearly in Figure 10, where the topology of the steady-state region in the $M_{WD} - \dot{M}$ plane is provided for the three metallicities considered. Reducing the metallicity, the steady-state burning region drops to smaller accretion rates and its extension is drastically decreased.

The consequences of our results for the final behavior of real low metallicity accretors are not easy to predict. On the one hand, as metallicity is decreased, the hydrogen-burning shell becomes hotter. This means that the underlying helium-burning layer is hotter and less degenerate when a helium-burning thermonuclear runaway is initiated. On the other hand, the fact that, for fixed core mass and accretion rate, the power of hydrogen-burning flash decreases as the metallicity is reduced suggests that low metallicity accretors may experience relatively mild helium shell flashes for a range of helium layer masses more extended than in the solar metallicity case.

However, as extensively discussed in Piersanti et al.(1999), in the mild pulse regime, there is a parameter region in which the effects of the hydrogen-burning shell on the helium layer are negligible, and a model with characteristics in theis region behaves as if pure helium is accreted. These models lead to a sub-Chandrasekhar explosion if the initial mass of the white dwarf and the accretion rate are within a given range (see Fig. 1 in Tornambé et al. 1998 for the solar metallicity case). Therefore, over the long term evolution, once the helium-burning layer becomes thermally decoupled from the hydrogen-burning shell, the accreted layer behaves in a way that depends only on the accretion rate and not on the metallicity.

Due to the prohibitively long computing time required, we have only partially studied the long term evolution of models accreting hydrogen-rich matter of metallicities $Z = 0.001$ and $Z = 0.0001$ at the mass-accretion rate of $\dot{M} = 10^{-8}\ M_{\odot}\,\mathrm{yr}^{-1}$. Both models show that the helium layer and the hydrogen-rich layer become decoupled as in the case of accretion of hydrogen-rich matter of solar metallicity (Piersanti et al. 1999). Such models will likely experience similar outcomes independent of the metallicity of the hydrogen-rich accreted matter.

On the basis of the results obtained so far, we suggest that, on lowering the metallicity, the area in the $M_{WD} - \dot{M}$ plane suitable for sub-Chandrasekhar dynamical outcomes is shifted toward slightly lower values of $\dot{M}$, remaining almost unchanged in extention, as indicated in Tornambé et al. (1998) for the solar metallicity. It has to be finally considered that metallicity could even affect other parameters of the binary system (as, for instance, initial white dwarf masses, accretion rates, etc) with the consequence that in the real world, this scenario could be also significantly changed.

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Fig. 1.— The track in the HR diagram of a white dwarf of mass $\sim 0.5236 M_\odot$ accreting hydrogen-rich matter of composition $Y = 0.28$, $Z = 0.02$ at the rate $\dot{M} = 10^{-8}\ M_\odot\,\mathrm{yr}^{-1}$. Evolution progresses in a counter clockwise fashion along the track from the reddest point (RP) to the bluest point (BP) and so on. The labels IC and EC indicate, respectively, the onset and offset of shell convection driven by a hydrogen shell flash. $\Phi_{\mathrm{H}} = L_{\mathrm{H}}/L_{\mathrm{s}}$ and $\Phi_{\mathrm{He}} = L_{\mathrm{He}}/L_{\mathrm{s}}$ where L_{H}, L_{He}, and L_{s} are respectively, the hydrogen-burning, helium-burning, and surface luminosities. In the plot we have also indicated selected specific models for which thermal and nuclear burning characteristics are displayed in Figs. 3 and 4 (see text).

Fig. 2.— The evolution during two successive pulse cycles of several characteristics of the model shown in Fig. 1. In panel (a), the dashed line gives the total mass of the model and the solid line gives the mass in the hydrogen-burning shell where the rate of nuclear energy generation ϵ_{H} is at a maximum. Panels (b) and (c) give, respectively, the temperature and density at the point where ϵ_{H} is at a maximum. In panel (d), $\Phi_{\mathrm{gr}} = L_{\mathrm{gr}}/L_{\mathrm{s}}$, L_{gr} is the rate of release of gravothermal energy, and L_{s} and Φ_{H} are defined in the caption of Fig. 1. Panel (e) gives the radius of the point where ϵ_{H} is at a maximum and panel (f) gives the radius of the surface.

Fig. 3.— The evolution of temperature profiles over the outer part of the accreting white dwarf which follows the track in the HR diagram given in Fig. 1. Every curve corresponds to one of the models indicated in Fig. 1 (see text).

Fig. 4.— The evolution of the profiles of the H-burning efficiency (ϵ_{n}) over the outer part of the accreting white dwarf which follows the track in the HR diagram given in Fig. 1. Every curve corresponds to one of the models indicated in Fig. 1, as described in the text.

Fig. 5.— The rate of energy generation at its maximum in the hydrogen-burning shell (panel

a) and the rate at which the center of the hydrogen-burning shell processes mass (panel b) along the high luminosity branch for the model with $M \sim 0.5236\ M_{\odot}$ and $\dot{M} = 10^{-8}$ $M_{\odot}\,\mathrm{yr}^{-1}$ for the solar metallicity case.

Fig. 6.— The rate at which the hydrogen-burning shell processes matter along the high luminosity branch for four different white dwarf masses as labelled. Hydrogen-rich matter with $Y = 0.28$ and $Z = 0.02$ is accreted at the rate $10^{-8}\ M_{\odot}\,\mathrm{yr}^{-1}$.

Fig. 7.— Evolution in the HR diagram during one hydrogen-pulse cycle for white dwarf models accreting hydrogen-rich matter with three different metallicities: $Z = 0.02, 0.001$ and 0.0001. The three models have almost the same mass ($M \sim 0.523 M_{\odot}$).

Fig. 8.— The same as in Figure 2, but for the cases Z=0.001 (solid line) and Z=0.0001 (heavy dashed line) (see text).

Fig. 9.— Evolution of the mass coordinate of the center of the hydrogen-burning shell (where the rate of energy generation is at a maximum) in models which accrete hydrogen-rich matter with metallicities of $Z = 0.02, 0.001$, and 0.0001 at the rate $\dot{M} = 10^{-8}\ M_{\odot}\,\mathrm{yr}^{-1}$.

Fig. 10.— The steady burning zone in the $M_{\mathrm{WD}} - \dot{M}$ plane for the three metallicity (Z=0.02, Z=0.001 and Z=0.0001), as obtained in the present work.

Table 1. Selected evolutionary and structural properties of the initial model and of the structure after it has experienced about 60 H-pulses.

	Initial model	**Model after 60 H-pulses**[1)]
Age (10^8 yr)	1.4488	1.4740
M ($M_\odot$)	0.5168	0.5235
$\log(L/L_\odot)$	-3.770	3.421
$\log(T_\mathrm{e})$	3.7479	5.324
$\log(R/R_\odot)$	-1.858	-1.416
$M_\mathrm{H-sh}$ ($M_\odot$)	0.5163	0.5232
$M_\mathrm{He-sh}$ ($M_\odot$)	0.4793	0.4793
$\log(T_\mathrm{H-sh})$	6.5666	7.7300
$\log(\rho_\mathrm{H-sh})$	3.8968	1.5815
$\log(T_\mathrm{He-sh})$	6.5823	7,7600
$\log(\rho_\mathrm{He-sh})$	5.2824	3.4481
$\log(T_\mathrm{c})$	6.6017	6.9355
$\log(\rho_\mathrm{c})$	6.3700	6.4091

[1)]The listed quantities refer to the model at the bluest point along the loop in the HR diagram.

Table 2. Minimum accretion rate for steady burning accretion as a function of white dwarf mass. The chemical composition of the accreted matter is: $Y = 0.28$, $Z = 0.02$. The location in the HR diagram of the bluest point along the track ($\log T_{\rm e}$, $\log L/L_{\odot}$) and $\Phi_{\rm H} = L_{\rm H}/L_{\odot}$ are also reported.

$M_{\rm WD}(M_{\odot})$	$\log(\dot{M})(M_{\odot}\,{\rm yr}^{-1})$	$\Phi_{\rm H}$	$\log T_{\rm e}$	$\log(L/L_{\odot})$
0.520	-7.585	0.939	5.325	3.421
0.540	-7.520	0.952	5.334	3.562
0.560	-7.493	0.938	5.358	3.519
0.581	-7.428	0.942	5.380	3.583
0.600	-7.378	0.938	5.401	3.634
0.620	-7.345	0.930	5.423	3.671
0.640	-7.336	0.914	5.444	3.688
0.660	-7.255	0.942	5.460	3.755
0.680	-7.236	0.935	5.479	3.777

Table 3. Maximum accretion rate for steady state burning and position in HR diagram as a function of white dwarf mass. Composition of accreted matter is the same as in Table 1. The discontinuity in $\log T_{\rm e}$ at $M_{\rm WD} > 0.6\ M_{\odot}$ is due to a different approach in the treatment of atmospheric layers adopted to prevent a huge expansion of the envelope.

$M_{\rm WD}$ ($M_{\odot}$)	$\log(\dot{M})$ ($M_{\odot}\,{\rm yr}^{-1}$)	$\Phi_{\rm H}$	$\log T_{\rm e}$	$\log(L/L_{\odot})$
0.520	-7.018	1.042	4.474	3.949
0.540	-6.988	1.044	4.544	3.978
0.560	-6.958	1.024	4.434	4.014
0.580	-6.917	1.031	4.340	4.055
0.600	-6.894	1.005	4.236	4.088
0.620	-6.859	1.004	4.642	4.124
0.640	-6.821	1.005	4.583	4.161
0.660	-6.807	1.006	4.896	4.174
0.680	-6.779	1.006	4.923	4.203

Table 4. The principal characteristics of the helium- and hydrogen-burning shells as a function of metallicity Z for models of mass $M \sim 0.5236\ M_\odot$ accreting hydrogen-rich matter at the rate $\dot{M} = 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$.

	$Z = 0.02$	$Z = 0.001$	$Z = 0.0001$
Period[a)]	1.24	2.39	3.27
$L_{\mathrm{H,max}}$[b)]	13.80	8.62	2.61
$L_{\mathrm{H,min}}$[c)]	0.394	0.650	1.450
$L_{\mathrm{He,max}}$[d)]	1.75	4.70	56.20
$\Delta M_{\mathrm{H,max}}$[e)]	1.76	3.45	5.04
$\Delta M_{\mathrm{H,min}}$[f)]	5.70	12.70	23.20
$\log(T_{\mathrm{H,min}})$[g)]	7.385	7.436	7.496
$\log(T_{\mathrm{H,max}})$[h)]	7.953	7.990	8.005
$\log(\rho_{\mathrm{H,min}})$[i)]	1.214	1.337	1.559
$\log(\rho_{\mathrm{H,max}})$[j)]	2.734	2.956	3.067

[a)]Time between two successive H-flash (10^4yr).

[b)]Maximum luminosity of H-shell ($10^5 L_\odot$).

[c)]Minimum luminosity of H-shell ($L_\odot$).

[d)]Maximum luminosity of He-shell ($10^{-6} L_\odot$).

[e)]Thickness in mass of H-shell at $L_{\mathrm{H,max}}$ ($10^{-4} M_\odot$).

[f)]Thickness in mass of H-shell at $L_{\mathrm{H,min}}$ ($10^{-5} M_\odot$).

[g)]Temperature of H-shell at $L_{\mathrm{H,min}}$

[h)]Temperature of H-shell at $L_{\mathrm{H,max}}$

[i)]Density of H-shell at $L_{\mathrm{H,min}}$

[j)]Density of H-shell at $L_{\mathrm{H,max}}$

Table 5. The minimum and maximum values of the accretion rate for which a white dwarf of mass $M \sim 0.5236\ M_{\odot}$ burns hydrogen as rapidly as it accretes it at different metallicities.

Z	$\dot{M}_{\rm low}$ ($10^{-8}\ M_{\odot}\,{\rm yr}^{-1}$)	$\dot{M}_{\rm high}$ ($10^{-8}\ M_{\odot}\,{\rm yr}^{-1}$)
0.0001	1.35	6.50
0.001	2.05	8.30
0.02	2.60	9.60

Table 6. Maximum accretion rate for steady state burning and position in the HR diagram as a function of white dwarf mass. The chemical composition of accreted matter is $Y = 0.28$ and $Z = 0.0001$ or $Z = 0.001$.

$M_{\rm WD}(M_\odot)$	$\log(\dot{M})(M_\odot\,{\rm yr}^{-1})$	$\Phi_{\rm H}$	$\log T_{\rm e}$	$\log(L/L_\odot)$
		$Z = 0.0001$		
0.520	-7.187	1.083	4.835	3.762
0.540	-7.142	1.125	4.890	3.791
0.560	-7.100	1.122	4.899	3.834
0.580	-7.058	1.062	4.863	3.900
0.591	-7.025	1.101	4.886	3.918
0.600	-7.005	1.104	4.892	3.937
		$Z = 0.001$		
0.520	-7.084	1.099	4.706	3.859
0.540	-7.060	1.073	4.704	3.894
0.560	-7.022	1.086	4.751	3.927
0.580	-6.978	1.084	4.731	3.971
0.590	-6.960	1.089	4.707	3.987

Table 7. Minimum accretion rate for steady state burning and position in the HR diagram as a function of white dwarf mass for two different assumptions on the metallicities of the accreted matter and the same He content.

$M_{WD}(M_{\odot})$	$\log(\dot{M})(M_{\odot}\,\mathrm{yr}^{-1})$	Φ_{H}	$\log T_{\mathrm{e}}$	$\log(L/L_{\odot})$
		$Z = 0.0001$		
0.520	-7.870	0.967	5.206	3.142
0.540	-7.795	0.966	5.235	3.204
0.560	-7.721	0.964	5.259	3.279
0.580	-7.639	0.963	5.284	3.362
0.591	-7.598	0.963	5.295	3.402
0.600	-7.519	0.968	5.304	3.479
		$Z = 0.001$		
0.520	-7.691	0.964	5.268	3.310
0.540	-7.631	0.965	5.279	3.369
0.560	-7.583	0.961	5.302	3.418
0.580	-7.510	0.961	5.325	3.491
0.590	-7.482	0.960	5.338	3.520